Oberlin High School
Mathematics Department
Grade 11 Matrices (Review) Notes for the Week of October 20 - 25, 2020

(See Your Grade 9 Note Book for Additional Notes)

Matrices

A **matrix** is a rectangular array of numbers enclosed by brackets. (The plural of matrix is **matrices.**)

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 7 & -1 \end{bmatrix}$ $\begin{bmatrix} 6 & -2 & -1 \end{bmatrix}$ $\begin{bmatrix} -5 \\ 3 \\ 10 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 \\ 3 & -9 \end{bmatrix}$ are all examples of matrices.

The numbers in a matrix are called the **elements** (or **entries)** of the matrix. The number of **rows** (horizontal) and the number of **columns** (vertical) determine the dimensions of the matrix . You always write the number of rows first and the number of columns second. In order, the dimensions of the above matrices are 3×2 (read 3 by 2), 1×4 , 3×1 and 2×2 .

A matrix with only one row (the second one above) is called a **row matrix.** If the matrix has only one column (the third one above) is a **colum matrix.** The last matrix above is a square matrix because the number of rows equals the number of columns.

If all of the elements of a matrix are zero, it is called a **zero matrix** .

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a 2×3 zero matrix, denoted $0_{2\times3}$.

One common use of matrices is for solving systems of linear equations . For this, you need to know about matrix row operations and the ident matrix .

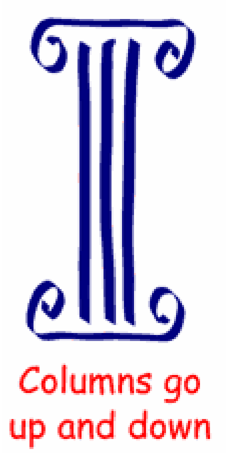
You can also do algebra with matrices -- that is, you can add them and subtract them , multiply them (if their dimensions are compatible), and even do a sort of division by finding their inverses (this only works for square matrices). In advanced mathematics, matrices are used to describe **linear transformations**.

Notation

A matrix is usually shown by a **capital letter** (such as A, or B)

Each entry (or "element") is shown by a **lower case letter** with a "subscript" of **row,column**:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{bmatrix}$$



Rows and Columns

So which is the row and which is the column?

- Rows go **left-right**
- Columns go **up-down**

To remember that rows come before columns use the word **"arc"**:

$$a_{r,c}$$

Example:

$$B = \begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

Here are some sample entries:

$b_{1,1} = 6$ *(the entry at row 1, column 1 is 6)*

$b_{1,3} = 24$ *(the entry at row 1, column 3 is 24)*

$b_{2,3} = 8$ *(the entry at row 2, column 3 is 8)*

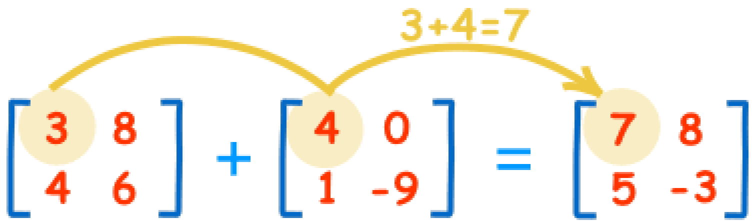
Adding & Subtracting

A matrix can only be added to (or subtracted from) another matrix if the two matrices have the same dimensions .

To add two matrices, just add the corresponding entries, and place this sum in the corresponding position in the matrix which results.

Adding

To add two matrices: add the numbers in the matching positions:



These are the calculations:

3+4=7   8+0=8

4+1=5   6−9=−3

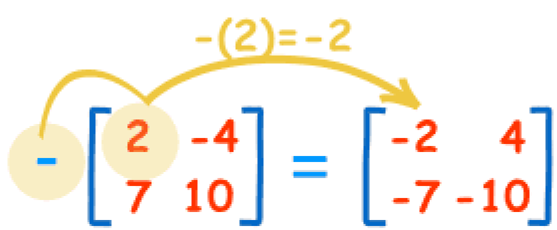
The two matrices must be the same size, i.e. the rows must match in size, and the columns must match in size.

Example: a matrix with **3 rows** and **5 columns** can be added to another matrix of **3 rows** and **5 columns**.

But it could not be added to a matrix with **3 rows** and **4 columns** (the columns don't match in size)

Negative

The negative of a matrix is also simple:



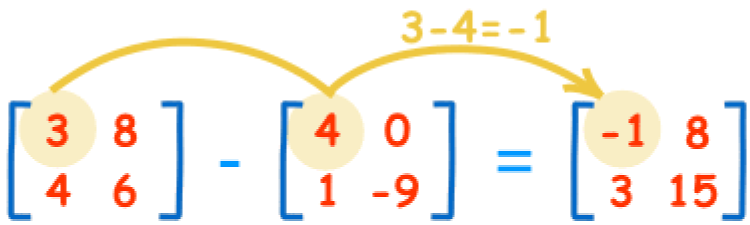
These are the calculations:

−(2)=−2   −(−4)=+4

−(7)=−7   −(10)=−10

Subtracting

To subtract two matrices: subtract the numbers in the matching positions:



These are the calculations:

3−4=−1 8−0=8

4−1=3 6−(−9)=15

*Note: subtracting is actually defined as the **addition** of a negative matrix: A + (−B)*

Example 1:

Add the matrices.

$$\begin{bmatrix} 1 & 5 \\ -4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 4 & -1 \end{bmatrix}$$

First note that both addends are 2×2 matrices, so we can add them.

$$\begin{bmatrix} 1 & 5 \\ -4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 1+2 & 5+(-1) \\ -4+4 & 3+(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix}$$

Example 2:

Solve:

$$\begin{pmatrix} 5 & 2 \\ 4 & 9 \\ 10 & -3 \end{pmatrix} + \begin{pmatrix} -11 & 0 \\ 7 & 1 \\ -6 & -8 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 4 & 9 \\ 10 & -3 \end{pmatrix} + \begin{pmatrix} -11 & 0 \\ 7 & 1 \\ -6 & -8 \end{pmatrix} = \begin{pmatrix} 5+(-11) & 2+0 \\ 4+7 & 9+1 \\ 10+(-6) & -3+(-8) \end{pmatrix}$$

$$= \begin{pmatrix} -6 & 2 \\ 11 & 10 \\ 4 & -11 \end{pmatrix}$$

Example 3:

Subtract.

$$\begin{bmatrix} 4 & 5 & 6 \\ 2 & 3 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

Subtract corresponding entries.

$$\begin{bmatrix} 4 & 5 & 6 \\ 2 & 3 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4-2 & 5-4 & 6-6 \\ 2-1 & 3-2 & 4-3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Example 4:
Solve:

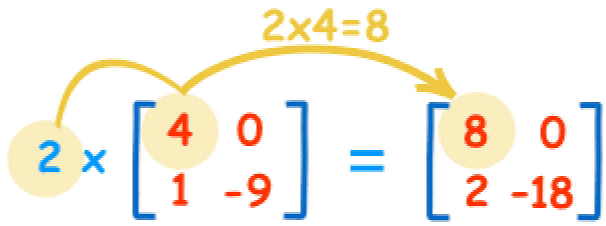
$$\begin{pmatrix} 7 & 3 \\ 5 & 9 \\ 11 & -2 \end{pmatrix} - \begin{pmatrix} -3 & 0 \\ 8 & 1 \\ -3 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 7 & 3 \\ 5 & 9 \\ 11 & -2 \end{pmatrix} - \begin{pmatrix} -3 & 0 \\ 8 & 1 \\ -3 & -4 \end{pmatrix} = \begin{pmatrix} 7-(-3) & 3-0 \\ 5-8 & 9-1 \\ 11-(-3) & -2-(-4) \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 3 \\ -3 & 8 \\ 14 & 2 \end{pmatrix}$$

Multiply by a Constant

We can multiply a matrix by a **constant** *(the value 2 in this case)*:



These are the calculations:

2×4=8   2×0=0

2×1=2   2×−9=−18

We call the constant a **scalar**, so officially this is called "scalar multiplication".

Multiplying by Another Matrix
To **multiply two matrices together** is a bit more difficult ... this will be covered in the next lesson

Dividing
And what about division? Well we **don't** actually divide matrices, we do it this way:

$$A/B = A \times (1/B) = A \times B^{-1}$$

where $\mathbf{B^{-1}}$ means the "inverse" of B.

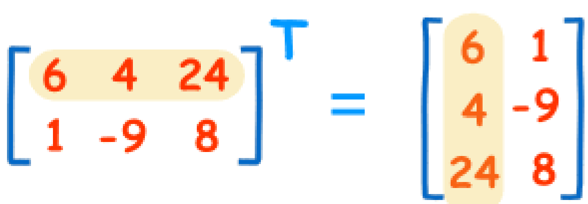
So we don't divide, instead we **multiply by an inverse**.

And there are special ways to find the Inverse, we will look at these in future lessons.

Transposing

To "transpose" a matrix, swap the rows and columns.

We put a "T" in the top right-hand corner to mean transpose:



Oberlin High School
Mathematics Department
Grade 11 Matrices (Review) Activity for the Week of October 20 - 25, 2020

Student Name: _______________ Teacher: __________________________

Class: ____________________ Date Submitted: ____________________

General Instructions

a) Solve all questions and **show working.**
b) Take a picture of your work and email to Oberlin.math@gmail.com to be graded.

1. Given that A = $\begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -4 & 2 \\ 0 & 3 \end{bmatrix}$

Find:

a) A + B *(2 marks)*
b) B – A *(2 marks)*
c) 5B *(1 mark)*
d) 3A – B *(3 marks)*
e) ½ A *(1 mark)*

2. What is the transpose of the matrix A in number 1? ***(See your notes for example)*** *(1 mark)*

TOTAL MARKS: 10