EXPRESSING A QUADRATIC FUNCTION IN ANOTHER FORM USING THE *h* AND *k* FORMULA

Example 1:

a. Express $2x^2 + 8x - 6$ in the form $a(x + h)^2 + k$, where a, h and k are real numbers.

Solution (Using a Formula):

a ***b*** ***c***

a) First identify the values of a, b and c of $2x^2 + 8x - 6$.

That is ***a*** $= 2$,

b $= 8$

c $= -6$

Second calculate the values of ***h*** and ***k***

h $= \frac{b}{2a}$

$= \frac{8}{2(2)}$

$= \frac{8}{4}$

$= 2$

k $= \frac{4ac - b^2}{4a}$

$= \frac{4(2)(-6) - 8^2}{4(2)}$

$= \frac{-48 - 64}{8}$

$= \frac{-112}{8}$

$= -14$

$a(x + h)^2 + k$

Therefore $2x^2 + 8x - 6 = 2(x + 2)^2 - 14$.

b. Using the answer from above, or otherwise, calculate

i. The minimum value of **$2x^2 + 8x - 6$** (***Solution: $y = -14$***)

ii. The value of x for which the minimum occurs (***Solution: $x = -2$*** **←Notice that the sign is changed for the *h* value. In other words $x = -h$**)

iii. The vertex or coordinates of the minimum point (***Solution: $(-2, -14)$***)

Note: the vertex is (-h,k) using the formula in part a

iv. The value of the y- intercept (***Solution: $y = -6$*** ***← If $x = 0$ in the equation $y = 2x^2 + 8x - 6$ then $y = -6$***)

Example 2:

2a. Express $-2x^2 - 3x + 4$ in the form $a(x + h)^2 + k$, where a, h and k are real numbers.

Solution (Using a Formula):

For $-2x^2 - 3x + 4$,

a $= -2$, ***b*** $= -3$ *and* ***c*** $= 4$.

h $= \frac{b}{2a}$

$= \frac{-3}{2(-2)}$

$= \frac{-3}{-4}$

$= \frac{3}{4}$

$$\mathbf{k} = \frac{4ac - b^2}{4a}$$

$$= \frac{4(-2)(4) - (-3)^2}{4(-2)}$$

$$= \frac{-32 - 9}{-8}$$

$$= \frac{-41}{-8} = \frac{41}{8} \text{ or } 5\,{}^{1}\!/_{8}$$

$$a(x + h)^2 + k$$

Therefore $-2x^2 - 3x + 4 = -2\left(x + \frac{3}{4}\right)^2 + \frac{41}{8}$

2b. Using the answer from above, or otherwise, calculate

v. The minimum value of $-2x^2 - 3x + 4$ (***Solution:*** $y = \frac{41}{8}$ **)**

vi. The value of x for which the minimum occurs (***Solution:*** $x = -\frac{3}{4}$ **←Notice that the sign is changed for the *h* value. In other words** $x = -h$)

vii. The vertex or coordinates of the minimum point (***Solution:*** $\left(-\frac{3}{4}, \frac{41}{8}\right)$ **or** $\left(-\frac{3}{4}, 5\,{}^{1}\!/_{8}\right)$

Note: the vertex is (-h,k) using the formula in part a

viii. The value of the y- intercept (***Solution:*** $y = 4$ ***← If*** $x = 0$ ***in the equation*** $y = -2x^2 - 3x + 4$ **then y = 4)**

ACTIVITY

a. Express $\mathbf{3x^2 - 8x + 2}$ in the form $a(x + h)^2 + k$, where a, h and k are real numbers.

b. Using the answer from above, or otherwise, calculate
   i. The minimum value of $\mathbf{3x^2 - 8x + 2}$
   ii. The value of x for which the minimum occurs
   iii. The coordinates of the minimum point
   iv. The value of the y- intercept